# Math 126 Autumn 2009
Work with lines and planes.

1. Decide by yourself whether each of the following is true or false. Compare answers with one or two neighbors, then confirm your answers by using pieces of paper and/or a desktop as models for planes, and pens and/or pencils as models for lines.

   FOR DETAILS SEE ATTACHED

   (a) Two lines perpendicular to the same plane are parallel. TRUE

   (b) Two lines parallel to the same plane are parallel. FALSE

   (c) Two planes perpendicular to the same (third) plane are parallel. TRUE

   (d) Two planes parallel to the same (third) plane are parallel. TRUE

   (e) Two lines perpendicular to the same (third) line are parallel. FALSE

   (f) Two lines parallel to the same (third) line are parallel. TRUE

   (g) Two planes either intersect or are parallel. TRUE

   (h) Two planes perpendicular to the same line are parallel. TRUE

   (i) Two planes parallel to the same line are parallel. FALSE

Use the following axes for questions 2 and 3.
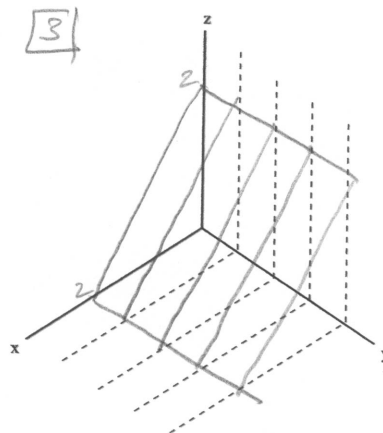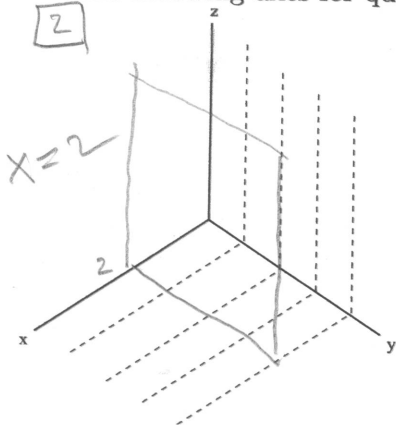
2. Find the equation and sketch the graph of a plane that is parallel to the yz-coordinate plane and contains the point $(2, 1, 3)$. How is this plane related to the other two coordinate planes, the xy-coordiante plane and the xz-coordinate plane?

   SEE ATTACHED

3. Graph the plane P given by the equation $x + z = 2$. → NORMAL VECTOR $\langle 1, 0, 1 \rangle$!

   SEE ATTACHED

   Is P parallel to any of the coordinate planes?
   Is P perpendicular to any of the coordinate planes?
   Is P parallel to any of the coordinate axes?
   Is P perpendicular to any of the coordinate axes?
   What fact about the equation for P immediately gives you the anwer to all of these questions?

WORKSHEET 2b NOTES AND SOLUTIONS

(1) (SEE PAGE 1 FOR ANSWERS)

NOTE: We defined

TWO LINES ARE PARALLEL ⟺ THEIR DIRECTION VECTORS ARE PARALLEL

OTHERWISE THEY EITHER <u>INTERSECT</u> OR ARE <u>SKEW</u>

TWO PLANES ARE PARALLEL ⟺ THEIR NORMALS ARE PARALLEL
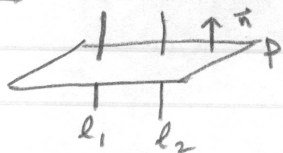TWO PLANES ARE PERPENDICULAR ⟺ THEIR NORMALS ARE PERPEN.
IF TWO PLANES ARE NOT PARALLEL,
THEN THEY MUST INTERSECT (IN A LINE).

SPECIFIC COMMENTS:
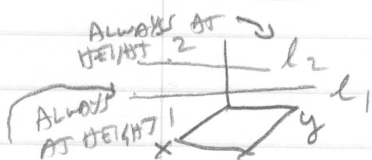
(a) TWO LINES PERPENDICULAR TO SAME PLANE ARE PARALLEL.
[TRUE!] LINE PERPENDICULAR TO PLANE ⟹ LINE DIRECTION VECTOR
AND PLANE NORMAL ARE PARALLEL.

ℓ_1 ℓ_2 $\vec{n}$ P

(b) TWO LINES PARALLEL TO THE SAME PLANE ARE PARALLEL.
[FALSE!] THE LINES COULD BE SKEW!

ALWAYS AT HEIGHT 2 ℓ_2
ALWAYS AT HEIGHT 1 ℓ_1

COUNTER-EXAMPLE
$\ell_1: x = 2t, y = t, z = 1$ } BOTH PARALLEL
$\ell_2: x = 7s, y = 3s, z = 2$ } TO xy-PLANE

AND ℓ_1 & ℓ_2 ARE <u>NOT</u> PARALLEL (THEY ARE SKEW)

(c) TWO PLANES PERPENDICULAR TO THE SAME PLANE ARE PARALLEL.
[TRUE!] $\vec{n_3}$ $\vec{n_2}$ $\vec{n_1}$ } P_1 and P_2 must be parallel.
P_3 P_1 P_2
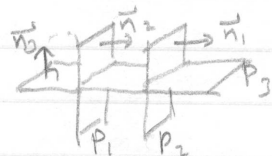
(d) TWO PLANES PARALLEL TO THE SAME PLANE ARE PARALLEL
[TRUE] $\vec{n_1}$ $\vec{n_2}$ $\vec{n_3}$ } $\vec{n_1}, \vec{n_2},$ and $\vec{n_3}$ are all parallel
P_1 P_2 P_3
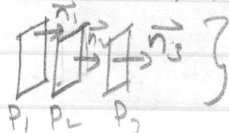
(e) TWO LINES PERPENDICULAR TO THE SAME LINE ARE PARALLEL.

FALSE! l_2 COULD BE SKEW
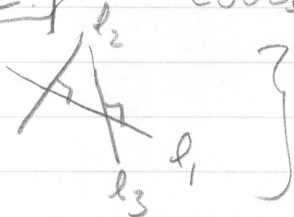
l_2 AND l_3 <u>DON'T</u> HAVE TO BE PARALLEL

l_2, l_1, l_3

(f) TWO LINES PARALLEL TO THE SAME LINE ARE PARALLEL

TRUE! $\vec{v}_1$, l_1, $\vec{v}_2$, l_2, $\vec{v}_3$, l_3 $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ are all parallel.
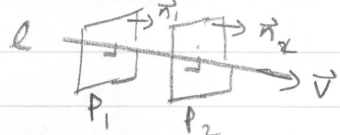
(g) TWO PLANES EITHER INTERSECT OR ARE PARALLEL.

TRUE!

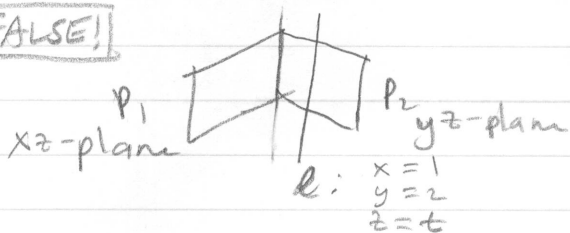(h) TWO PLANES PERPENDICULAR TO THE SAME LINE ARE PARALLEL.

TRUE! l $\vec{n}_1$ $\vec{n}_2$ $\vec{v}$ P_1 P_2 $\vec{n}_1$, $\vec{m}_2$, and $\vec{v}$ are all parallel.

(i) TWO PLANES PARALLEL TO THE SAME LINE ARE PARALLEL.

FALSE!

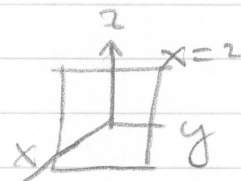P_1 xz-plane P_2 yz-plane l: $x=1$, $y=2$, $z=t$

P_1 & P_2 are parallel to l but P_1 and P_2 are not parallel

② PARALLEL TO yz-PLANE AND THROUGH (2,1,3) WOULD BE GIVEN BY $X=2$ IT IS PERPENDICULAR TO THE xy-plane and xz-plane.

z $x=2$ x y

③ $x+z=2 \Rightarrow z=2-x \longrightarrow$

z (0,0,2)   (0,1,2) y (2,0,0)   (2,1,0) x

IT IS <u>NOT</u> PARALLEL TO ANY COORD. PLANES.
IT IS PERPENDICULAR TO THE XZ-PLANE.
IT IS PARALLEL TO THE y-AXIS.
IT IS <u>NOT</u> PERPENDICULAR TO ANY COORD. AXIS.
THE NORMAL VECTOR IS $\langle 1, 0, 1 \rangle$!
(Also contains x and z and no y)