

# Math 307 Final Equation Sheet

- $ay'' + by' + cy = 0$ ,  $ar^2 + br + c = 0$ ,  $\{e^{r_1 t}, e^{r_2 t}\}$ ,  $\{e^{rt}, te^{rt}\}$ ,  $\{e^{\lambda t} \cos(\mu t), e^{\lambda t} \sin(\mu t)\}$ .
- $y = y_h + y_p = c_1 y_1 + c_2 y_2 + y_p$
- $y = u y_1$ ,  $y = y_1 \left( \int \frac{y_1 e^{\int p(t) dt} g(t) dt + c}{y_1^2 e^{\int p(t) dt}} dt + d \right)$
- $y_p = u_1(t)y_1 + u_2(t)y_2$ ,  $u_1(t) = - \int \frac{g(t)y_2(t)}{W(y_1, y_2)} dt$ ,  $u_2(t) = \int \frac{g(t)y_1(t)}{W(y_1, y_2)} dt$ ,  $W(y_1, y_2) = y_1 y_2' - y_1' y_2$ .
- $A \cos(\omega t) + B \sin(\omega t) = R \cos(\omega t - \delta)$ ,  $A = R \cos \delta$ ,  $B = R \sin \delta$ ,  $R = \sqrt{A^2 + B^2}$ ,  $\delta = \text{atan2}(B, A)$ ,  $\text{atan2}(B, A)$  is  $\arctan(B/A)$  if  $A > 0$  and is  $\arctan(B/A) + \pi$  if  $A < 0$ .
- $my'' + \gamma y' + ky = F(t)$ ,  $mg = kL$
- $\gamma^2 - 4mk$  vs.  $0$ ,  $\omega_0 = \sqrt{\frac{k}{m}}$ ,  $\lambda = -\frac{\gamma}{2m}$ ,  $\mu = \sqrt{\frac{k}{m} - \frac{\gamma^2}{4m^2}} = \omega_0 \sqrt{1 - \frac{\gamma^2}{4km}}$ .

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
$af(t) + bg(t)$	$aF(s) + bG(s)$
1	$\frac{1}{s}$
$e^{at}$	$\frac{1}{s-a}$
$e^{at} f(t)$	$F(s-a)$
$t^n$	$\frac{n!}{s^{n+1}}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$\sin(bt)$	$\frac{b}{s^2 + b^2}$
$\cos(bt)$	$\frac{s}{s^2 + b^2}$
$e^{at} \sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$
$e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$
$\delta(t)$	1
$\delta(t-c)$	$e^{-cs}$
$u_c(t)$	$\frac{e^{-cs}}{s}$
$u_c(t)f(t)$	$e^{-cs} \mathcal{L}\{f(t+c)\}(s)$
$u_c(t)f(t-c)$	$e^{-cs} F(s)$