Your Preferred Name

Student ID #

- Do not open this exam until you are told to begin. You will have 50 minutes for the exam.
- Check that you have a complete exam. There are 5 questions for a total of 44 points.
- You are allowed to have one handwritten note sheet. An **equation sheet** is provided on the last page. No calculators are allowed.
- Cheating will result in a zero and be reported to the Dean's Academic Conduct Committee.
- **Show all your work**. Unless explicitly stated otherwise in a particular question, if there is no work supporting your answer, you will not receive credit for the problem. If you need more space to answer a question, continue on the back of the page, and indicate that you have done so.

Question	Points	Score
1	9	
2	9	
3	9	
4	7	
5	10	
Total:	44	

1. Consider the differential equation

$$(2-t)y = \frac{e^t}{t(t^2+1)} - ty'.$$

   (a) (2 points) Check all that apply. The differential equation is...
   ○ Autonomous ○ Linear ○ Separable
   ○ First order ○ Second order ○ Constant-coefficient

   (b) (5 points) Find the general solution of the differential equation. You may assume $t > 0$.

   (c) (2 points) Find the particular solution with $y(1) = 0$. (If you did not solve (b), you may use a reasonable guess instead.)

2. (a) (4 points) Find an explicit solution of the differential equation

$$y' = \frac{1 - y^2}{2y}.$$

(b) (5 points) **Classify** the equilibrium solutions of the differential equation

$$y' = (e^{1-y} - e^2)y$$

as stable, semistable, or unstable. Draw the **phase line** and some **sample solutions**. If $y(0) = -1/2$, **what is** $\lim_{t\to\infty} y(t)$?

3. (a) (3 points) Given

$$y' = ty + y^2, \qquad \text{and} \qquad y(0) = 1,$$

estimate $y(3)$ using Euler's method with $h = 1$.

Step k	t_k	y_k	$f(t_k, y_k)$

(b) (3 points) Determine the largest interval on which the IVP

$$\cos(t/2)y' - \sin(t/2)y = \ln|t+1|, \qquad y(0) = 5$$

is guaranteed to exist. *Do not attempt to solve* the differential equation.

(c) (3 points) Compute the second Picard iterate $\phi_2(t)$ for the IVP

$$y' = y^3 + 14t, \qquad y(0) = 0.$$

4. You are enjoying a cool 32 ounce drink on a warm spring day. You drink through a straw at a constant rate of 1 ounce per minute. Sadly, liquid is dripping through a hole in the bottom of the cup, losing liquid at a rate of 1/10th of an ounce per minute for each ounce of liquid remaining.

   (a) (3 points) **Write down** (but do not solve) an initial value problem for the volume $V(t)$ of liquid left after t minutes.

   (b) (4 points) The outside temperature is 70° F, and the drink is initially 40° F. The drink warms as a rate of 0.5° F per minute per degree of temperature difference. **Write down** (but do not solve) an initial value problem for the temperature $T(t)$ of the liquid after t minutes. **What is** $\lim_{t\to\infty} T(t)$?

5. (a) (3 points) Compute the **Wronskian** of $\{y_1, y_2\} = \{\cos e^t, \sin e^t\}$. In fact, the differential equation

$$y'' = y' - e^{2t} y$$

has $\{y_1, y_2\}$ as a **fundamental solution set**—what does this mean for applications?

(b) (3 points) Find the general solution of the differential equation

$$y'' + 10y' + 25y = 0.$$

(c) (4 points) Find two different **fundamental solution sets** of the differential equation

$$y'' + 4y' + 13y = 0.$$

What can you say about the **limiting behavior** $\lim_{t\to\infty} y$?

Equation Sheet

- $y' = f(t, y)$
- $y' = f(t)g(y)$
- $\int \frac{dy}{g(y)} = \int f(t)\,dt$
- $y' + p(t)y = g(t)$, $y(t_0) = y_0$
- $\mu(t) = e^{\int p(t)\,dt}$
- $y = \frac{1}{\mu(t)}(\int \mu(t)g(t)\,dt + c)$
- $y' = ry \pm k$, $T' = -k(T - T_S)$, $mv' = \pm mg - kv$
- $t_{n+1} = t_n + h$, $y_{n+1} = y_n + f(t_n, y_n)h$
- $\phi_0(t) = 0$, $\phi_{n+1}(t) = \int_0^t f(s, \phi_n(s))\,ds$
- $y'' = f(t, y, y')$
- $y'' + p(t)y' + q(t) = g(t)$, $y(t_0) = y_0$, $y'(t_0) = y_0'$
- $ay'' + by' + cy = 0$
- $ar^2 + br + c = 0$
- $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $r = \lambda \pm i\omega$
- $W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_1' y_2$
- $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$, $y = c_1 e^{rt} + c_2 t e^{rt}$, $y = d_1 e^{\lambda t}\cos(\omega t) + d_2 e^{\lambda t}\sin(\omega t)$