Your Name

Student ID #

1. Compute $\int x \cos x \, dx$.

Solution: Use integration by parts. Let $u = x$, $dv = \cos x \, dx$, so $du = dx$ and $v = \sin x$, giving

$$\int x \cos x \, dx = x \sin x - \int \sin x \, dx = x \sin x + \cos x + c.$$

2. Compute $\int \frac{t^2}{t^2-1} \, dt$.

Solution: Use partial fractions. The numerator's degree is too big, so divide it off by noting that $t^2 = (t^2 - 1)(1) + 1$, so that $\frac{t^2}{t^2-1} = 1 + \frac{1}{t^2-1}$. Now $t^2 - 1 = (t-1)(t+1)$, so write

$$\frac{1}{t^2 - 1} = \frac{A}{t-1} + \frac{B}{t+1}.$$

We get $A = 1/2$, $B = -1/2$. Hence

$$\begin{aligned} \int \frac{t^2}{t^2-1} \, dt &= \int \left(1 + \frac{1}{2}\frac{1}{t-1} - \frac{1}{2}\frac{t}{t+1}\right) dt \\ &= t + \frac{1}{2} \ln |t-1| - \frac{1}{2} \ln |t+1| + c. \end{aligned}$$

3. Solve the differential equation

$$y' = \cos x \sec y, \qquad y(0) = \pi/4.$$

Is your solution **implicit** or **explicit**?

Solution: The equation is separable. Separating gives

$$\cos y \, dy = \cos x \, dx.$$

Integrating gives

$$\sin y = \sin x + c.$$

Using the initial condition gives $c = \sin \pi/4 = \sqrt{2}/2$. The solution is then

$$\sin y = \sin x + \frac{\sqrt{2}}{2},$$

which is **implicit** since we haven't actually solved for y. (We could apply arcsin to both sides to get an explicit solution. This is a little sketchy since we only have $\arcsin(\sin(y)) = y$ for $-\pi/2 \leq y \leq \pi/2$, so if the initial condition had been instead $y(0) = 3\pi/4$ we'd have to tweak things. We'll typically ignore such subtleties.)