Your Preferred Name

Student ID #

1. Solve

$$y' = t^{1-t} - (1 + \ln t)y, \qquad y(1) = \frac{1}{2}$$

where $t > 0$.

(Hint: $e^{t \ln t} = t^t$.)

Solution: Use integrating factors. Integration by parts gives

$$\int (1 + \ln t)\, dt = t \ln t,$$

so the integrating factor is

$$\mu(t) = e^{t \ln t} = t^t.$$

Hence

$$y = t^{-t} \left(\int t^{t+1-t}\, dt + c \right) = t^{-t} \left(\frac{t^2}{2} + c \right).$$

At $t = 1$ we get $\frac{1}{2} = \frac{1}{2} + c$, so $c = 0$ and the answer is

$$y(t) = \frac{t^{2-t}}{2}.$$

2. Consider the differential equation

$$A'(x) = 0.3A(x) - 50, \qquad A(0) = 100.$$

Describe a physical situation modeled by this differential equation. Be sure to include the following: the units of x, A, A'; and physical meanings for the constants $0.3, 50, 100$. **Do not solve the DE.**

Solution: There are several models we discussed. A convenient one is repayment of a loan to a loan shark. x could be months, A could be dollars, so A' would be dollars per month. 0.3 or 30% would be the monthly rate of interest (compounded continuously), 50 would be the monthly repayment amount, and 100 would be the initial loan amount.