Your Preferred Name

Student ID #

Consider the following initial value problem:

$$y'' - 8y' + 15y = 15^2 \cdot t = 225t, \qquad y(0) = 8, y'(0) = 17.$$

1. (**Do not solve the IVP yet.**) Which of the following techniques could be used to solve this IVP? For those that apply, fill in the blanks.
   - ○ separable equation, with $f(y) =$ ________, $g(t) =$ ________
   - √ **variation of parameters, with** $y_1 =$ $e^{3t}$, $y_2 =$ $e^{5t}$
   - ○ integrating factors, with $\mu(t) =$ ________
   - ○ autonomous equation analysis, with $f(y) =$ ________
   - √ **reduction of order, with** $y_1 =$ $e^{3t}$ **or** $e^{5t}$
   - √ **undetermined coefficients, with** $Y =$ $At + B$

2. Pick one of the above techniques and find the **general solution** of the above differential equation.

**Solution:** Undetermined coefficients is likely to be fastest. Since $Y' = A$, $Y'' = 0$, the DE becomes

$$0 - 8(A) + 15(At + B) = 15At + (15B - 8A) = 15^2 t + 0.$$

Hence $15A = 15^2$ so $A = 15$ and $15B - 8A = 0$ so $B = 8$. Adding the homogeneous solutions then gives

$$y(t) = c_1 e^{3t} + c_2 e^{5t} + 15t + 8.$$

3. Solve the above initial value problem.

**Solution:** We compute

$$y(0) = c_1 + c_2 + 8 = 8,$$

so $c_1 = -c_2$. We also have

$$y'(0) = 3c_1 + 5c_2 + 15 = 17,$$

so $2c_2 = 2$. The solution is then

$$y(t) = -e^{3t} + e^{5t} + 15t + 8.$$