Your Preferred Name

Student ID #

Consider the following initial value problem:

$$y'' = u_1(t) \cdot \cos(t-1), \qquad y(0) = 0, y'(0) = 0.$$

(Take $t \geq 0$.)

1. Compute the Laplace transform of the right-hand side of the above differential equation.

**Solution:** From the identity $\mathcal{L}\{u_c(t)f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\}$, we have

$$\mathcal{L}\{u_1(t) \cdot \cos(t-1)\} = e^{-s}\mathcal{L}\{\cos(t)\} = e^{-s}\frac{s}{s^2+1}.$$

2. Compute

$$\mathcal{L}^{-1}\left(\frac{e^{-s}}{s(s^2+1)}\right).$$

(Hint: partial fractions.)

**Solution:** We have

$$\frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1},$$

so $1 = A(s^2+1) + (Bs+C)s = (A+B)s^2 + Cs + A$. Equating coefficients, $A = 1$, $C = 0$, $B = -1$. Now

$$\begin{aligned}\mathcal{L}^{-1}\left(\frac{e^{-s}}{s(s^2+1)}\right) &= \mathcal{L}^{-1}\left(\frac{e^{-s}}{s} - e^{-s}\frac{s}{s^2+1}\right)\\ &= u_1(t) \cdot 1 - u_1(t) \cdot \cos(t-1).\end{aligned}$$

3. Solve the above initial value problem. **What is $y(\pi+1)$?**

**Solution:** Using the initial conditions, we have $\mathcal{L}\{y''\} = s^2\mathcal{L}\{y\}$, so by (1)

$$\mathcal{L}\{y\} = \frac{e^{-s}}{s(s^2+1)}.$$

Thus $y$ is just the answer to (2),

$$y = u_1(t) \cdot 1 - u_1(t) \cdot \cos(t-1)$$

At $t = \pi + 1$ this is

$$y(\pi+1) = 1 - \cos(\pi) = 2.$$