Math 308 H **Midterm 2** **Winter 2015**

Your Name

Student ID #

- Do not open this exam until you are told to begin. You will have 50 minutes for the exam.
- Check that you have a complete exam. There are 4 questions for a total of 50 points.
- You are allowed to have one single sided, handwritten note sheet. Calculators are not allowed.
- Cheating will result in a zero and be reported to the Dean's Academic Conduct Committee.
- **Show all your work**. With the exception of True/False questions, if there is no work supporting your answer, you will not receive credit for the problem. If you need more space to answer a question, continue on the back of the page, and indicate that you have done so.

Question	Points	Score
1	16	
2	12	
3	11	
4	11	
Total:	50	

1. (16 points) True/False and short answer. No justification is necessary for the True/False questions.

   (a) If A and B are matrices such that $AB = C$, and C is invertible, then A and B are invertible.

   ○ True ○ False

   (b) If A is a 4×6 matrix, then the maximum value of $\text{rank}(A)$ is 6.

   ○ True ○ False

   (c) If A is an $n \times m$ matrix such that $A^T\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b} \in \mathbb{R}^m$, then $\text{row}(A) = \mathbb{R}^m$.

   ○ True ○ False

   (d) If A is an invertible $n \times n$ matrix, then $\text{nullity}(A) = 0$.

   ○ True ○ False

   (e) If A is a singular matrix that is row equivalent to B, then $\det(A) = \det(B)$.

   ○ True ○ False

   (f) Give an example of a matrix A such that $\text{rank}(A) < \text{nullity}(A)$.

   (g) Find the determinant of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 0 \\ -2 & -3 & 0 \end{bmatrix}$.

   (h) Is A (the same A as in part (g)) invertible? If so, find A^{-1}. If not, write one column of A as a linear combination of the others.

2. Let A be the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 2 & 1 & 1 & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix}.$$

(a) (4 points) Find a basis for the nullspace of A.

(b) (4 points) Find a basis for the column space of A.

(c) (4 points) Define a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ by $T(\mathbf{x}) = A\mathbf{x}$. Is T onto or one-to-one? Justify your answer.

3. Let $\mathbf{u}_1 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$, and let $S = \text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$.

   (a) (4 points) Find a matrix A such that the nullspace of A is equal to S.

   (b) (4 points) Extend $\{\mathbf{u}_1, \mathbf{u}_2\}$ to a basis for $\mathbb{R}^3$ (i.e., find another vector v such that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{v}\}$ is a basis for $\mathbb{R}^3$).

   (c) (3 points) Suppose $T : \mathbb{R}^6 \to \mathbb{R}^3$ is a linear transformation such that the range of T is equal to S. What is the dimension of the kernel of T? Justify your answer.

4. Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

   (a) (2 points) Explain why $\{\mathbf{u}_1, \mathbf{u}_2\}$ is a basis for $\mathbb{R}^2$.

   (b) (4 points) Write the vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ in terms of this basis (i.e., write $\mathbf{x}$ as a linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$). Your answer should involve x_1 and x_2.

   (c) (5 points) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $T(\mathbf{u}_1) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ and $T(\mathbf{u}_2) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. Find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$. (Hint: use your answer to part (b) to find a formula for $T(\mathbf{x})$).