Your Name

Student ID #

- Do not open this exam until you are told to begin.
- You will have until 11:20 to complete this exam; there are 6 questions for a total of 48 points.
- Graphing calculators and your textbook are not allowed. You may have one page of notes, double sided and handwritten, but this page of notes may not contain any proofs. Make sure your notes have your name on them and turn them in with your test.
- **Show your work**.
- Answer in the spaces provided; if you run out of room for an answer, continue on the back of the page and indicate that you have done so.
- Ambiguous or otherwise unreadable answers will be marked incorrect. So write clearly, erase fully, and provide only one answer to each question.
- This exam has 6 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	8	
2	8	
3	8	
4	8	
5	8	
6	8	
Total:	48	

1. (8 points) Select true or false for each of the following questions. There is no need to show any work.

   (a) A system of equations with more equations than variables has no solutions.

   ◯ True ◯ False

   (b) If A is an $n \times n$ matrix such that A^{-1} exists, then A and A^{-1} are row equivalent.

   ◯ True ◯ False

   (c) if A is an $n \times n$ matrix and $\mathcal{N}(A) = \overline{0}$, then A is nonsingular.

   ◯ True ◯ False

   (d) If $S = \{\overline{v}_1, \dots, \overline{v}_p\}$ is a set of vectors such that one vector can be written as a linear combination of the remaining vectors in S, then S is linearly dependent.

   ◯ True ◯ False

   (e) If A is any matrix whose columns are linearly independent, then A has an inverse.

   ◯ True ◯ False

   (f) If W is a subspace of $\mathbb{R}^n$, and $B = \{\overline{v}_1, \dots, \overline{v}_n\}$ is a basis for $\mathbb{R}^n$, then some subset of the vectors in B form a basis for W.

   ◯ True ◯ False

   (g) If $S = \{\overline{v}_1, \overline{v}_2, \overline{v}_3\}$ is a linearly independent set of vectors in $\mathbb{R}^3$, then any vector v in $\mathbb{R}^3$ can be written as a linear combination of the vectors in S.

   ◯ True ◯ False

   (h) If A and B are matrices such that BA is defined, then any vector $\overline{v}$ in $\mathcal{N}(A)$ is also in $\mathcal{N}(BA)$.

   ◯ True ◯ False

2. (8 points) For each set S of vectors given below, determine if S is a spanning set for $\mathbb{R}^3$. If it is **not** a spanning set, give an example of a vector $\overline{v}$ in $\mathbb{R}^3$ that **cannot** be written as a linear combination of the vectors in S.

(a)

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 7 \end{bmatrix} \right\}$$

(b)

$$S = \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \right\}$$

3. (8 points) Determine if the following matrices are nonsingular and find their inverse, if it exists. If it does not exist, express one column of the matrix as a linear combination of the remaining columns.

   (a)

$$A = \begin{bmatrix} 2 & 5 \\ 4 & 11 \end{bmatrix}$$

   (b)

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 2 & -4 & -10 \\ -1 & 3 & 9 \end{bmatrix}$$

4. (8 points) For each of the following subsets W, either prove that W is a subspace of $\mathbb{R}^3$ by checking the required conditions or prove that it is not a subspace by providing a counterexample to one of those conditions.

   (a)
$$W = \left\{ \overline{x} : \overline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},\ x_1 \leq x_2 \right\}$$

   (b)
$$W = \left\{ \overline{x} : \overline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},\ 3x_1 - 2x_2 + 2x_3 = 0 \right\}$$

5. (8 points) Let A be the matrix below whose reduced echelon form, ref(A), is also given:

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & -2 & 5 & 4 \\ 1 & -1 & 0 & 7 \end{bmatrix} \qquad \text{ref}(A) = \begin{bmatrix} 1 & -1 & 0 & 7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for $\mathcal{N}(A)$.

(b) Find a basis for $\mathcal{R}(A)$.

6. (8 points) Prove the following theorem:

   Theorem. Let W be a subspace of $\mathbb{R}^n$. If $B = \{\overline{v}_1, \dots, \overline{v}_p\}$ is a basis for W, then any vector $\overline{x}$ in W can be expressed uniquely in terms of the basis B. That is, there are unique scalars $a_1, \dots, a_p$ such that

   $$\overline{x} = a_1\overline{v}_1 + \cdots + a_p\overline{v}_p.$$