Math 308I Spring 2014
Final Exam
June 9, 2014

Your Name	Your Signature

Student ID #

Problem	Points	Possible
1		20
2		20
3		9
4		6
5		5
6		5
7		5
8		5
9		5
Total		80

- No books allowed. You may use one $8\frac{1}{2} \times 11$ sheet of notes.
- Do not share notes.
- In order to receive credit, you must show your work and explain your reasoning.
- **Place a box around $\boxed{\textbf{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the backs of the pages and indicate to the grader where to find your work.
- Raise your hand if you have a question or need more paper.

Don't open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

1. (20 points) Short answer questions. *No explanation of answers needed for this problem only. Be sure to explain your answers and show your work on all other problems!*

   (a) True or False: If A, B, and X are invertible matrices such that $XA = B$, then $X = A^{-1}B$.

   (b) True or False: If A is a square matrix whose columns add up to the zero vector, then A is invertible.

   (c) True or False: If A is an invertible matrix, then A and A^T have the same null space.

   (d) True or False: If A is a 3×5 matrix such that $(\text{row}\,(A))^{\perp} = \mathbb{R}^5$, then A must be the zero matrix.

   (e) True or False: If an $n \times n$ matrix has n distinct eigenvalues, then it must be diagonalizable.

   (f) True or False: The transformation $T : \mathbb{R}^5 \to \mathbb{R}^5$ defined by $T(\vec{x}) = -\vec{x}$ is a linear transformation.

   (g) True or false: The transformation $T : \mathbb{R}^4 \to \mathbb{R}^1$ defined by $T(\vec{x}) = \|\vec{x}\|$ is a linear transformation.

   (h) If A is a 3×5 matrix, what are the possible values of nullity(A)?

   (i) Let $\vec{a_1}$, $\vec{a_2}$, $\vec{a_3}$ be linearly independent vectors in $\mathbb{R}^7$, and let $A = [\vec{a_1}\ \vec{a_2}\ \vec{a_3}]$. What are the possible values for the rank of A?

   (j) Let S be a subspace of $\mathbb{R}^n$, and let $\vec{v}$ and $\vec{u}$ be vectors in $\mathbb{R}^n$. If $\text{proj}_S\vec{v} = \vec{u}$, what is $\text{proj}_S\vec{u}$?

2. (20=6+2+8+4 points) Let $A = \begin{bmatrix} -5 & -6 & 3 \\ 3 & 4 & -3 \\ 0 & 0 & -2 \end{bmatrix}$.

(a) Compute the characteristic polynomial of A.

(b) Find all of the eigenvalues of A.

(c) Find a basis for each of the eigenspaces of A.

(d) Is A diagonalizable? If yes, find P and D; if not, explain why.

3. (9=4+5 points) Let $\vec{x} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ and $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.

(a) Find the coordinate vector $[\vec{x}]_{\mathcal{B}}$ of $\vec{x}$ with respect to $\mathcal{B}$.

(b) Assume that $\mathcal{C}$ is another basis for $\mathbb{R}^2$, and that the change of basis matrix from $\mathcal{C}$ to $\mathcal{B}$ is $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$. Find the coordinate vector $[\vec{x}]_{\mathcal{C}}$ of $\vec{x}$ with respect to $\mathcal{C}$.

4. (6 points) Find a basis for $W^{\perp}$ if

$$W = \operatorname{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix} \right\}.$$

5. (5 points) Find the coordinate vector $[\vec{v}]_{\mathcal{B}}$ of $\vec{v} = \begin{bmatrix} 7 \\ -3 \\ 2 \end{bmatrix}$ with respect to *orthogonal* basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\} \text{ of } \mathbb{R}^3.$$

6. (5 points) Compute $\det A^5$ if $A = PDP^{-1}$, where

$$P = \begin{bmatrix} 3 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & -1 & -1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

7. (5 points) Show that if λ is an eigenvalue of an $n \times n$ matrix C, then λ^3 is an eigenvalue of C^3.

8. (5 points) Explain why a square matrix that has two equal rows, must have 0 as one of its eigenvalues.

9. (5 points) Let A and B be 3×4 matrices. Show that $W = \{\vec{x} \in \mathbb{R}^4 \ : \ A\vec{x} = B\vec{x}\}$ is a subspace of $\mathbb{R}^4$.