

Your Preferred Name

Student ID #

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- Do not open this quiz until you are told to begin. You will have 30 minutes for the quiz.
- Check that you have a complete quiz. There are 3 questions for a total of 28 points.
- You are allowed to have one index card of handwritten notes (both sides). Only basic non-graphing scientific calculators are allowed, though you should not need one.
- Cheating will result in a zero and be reported to the Dean's Academic Conduct Committee.
- **Show all your work.** Unless explicitly stated otherwise in a particular question, if there is no work supporting your answer, you will not receive credit for the problem. If you need more space to answer a question, continue on the back of the page, and indicate that you have done so.

Question	Points	Score
1	12	
2	7	
3	9	
Total:	28	

1. Consider the vectors

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \vec{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \vec{u}_2 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad \vec{u}_3 = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}.$$

- (a) (6 points) Determine if \vec{v} is in the span of $\vec{u}_1, \vec{u}_2, \vec{u}_3$.
If so, write \vec{v} as an explicit linear combination of the other vectors.

- (b) (3 points) Are $\vec{u}_1, \vec{u}_2, \vec{u}_3$ linearly dependent? Explain.

- (c) (3 points) Are $\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{v}$ linearly dependent? Explain.

2. Consider the function $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} \int_0^\pi (x_1\theta + x_2) \cos \theta \, d\theta \\ \int_0^\pi (x_1\theta + x_2) \sin \theta \, d\theta \end{bmatrix}$$

(Hints: $\int \theta \cos \theta \, d\theta = \cos \theta + \theta \sin \theta + C$ and $\int \theta \sin \theta \, d\theta = \sin \theta - \theta \cos \theta + C$.)

(a) (4 points) Is T a linear transformation? Justify your answer by verifying the relevant properties or by giving an explicit example where they fail.

(b) (3 points) If T is linear, find the matrix of T . If T is not linear, compute $T(\vec{e}_1)$ and $T(\vec{e}_2)$.

3. Give examples matching the following specifications. You do not need to justify your answers.

(a) (3 points) Three vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ in \mathbb{R}^4 which are linearly independent and where no coordinate is 0.

(b) (3 points) A linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ where $[T] \neq I$ yet $[T]^2 = I$.

(c) (3 points) Matrices A and B such that

$$AB = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \quad \text{and} \quad BA = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$