Your Preferred Name

Student ID #

- Do not open this quiz until you are told to begin. You will have 30 minutes for the quiz.
- Check that you have a complete quiz. There are 3 questions for a total of 29 points.
- You are allowed to have one index card of handwritten notes (both sides). Only basic non-graphing scientific calculators are allowed, though you should not need one.
- Cheating will result in a zero and be reported to the Dean's Academic Conduct Committee.
- **Show all your work**. Unless explicitly stated otherwise in a particular question, if there is no work supporting your answer, you will not receive credit for the problem. If you need more space to answer a question, continue on the back of the page, and indicate that you have done so.

Question	Points	Score
1	9	
2	10	
3	10	
Total:	29	

1. (9 points) Below is a list of definitions or equivalent conditions. Clearly write the number of the corresponding definition or equivalent condition next to each given term. Some numbers will not be used.

   (a) ___**9**___ dimension

   (b) ___**2**___ basis

   (c) ___**1**___ subspace

   (d) ___**7**___ row space

   (e) ___**8**___ invertible

   (f) ___**3**___ kernel

   (g) ___**5**___ column space

   (h) ___**6**___ range

   (i) ___**11**___ nullity

   1. a subset of $\mathbb{R}^n$ which contains $\vec{0}$ and is closed under linear combinations
   2. a linearly independent spanning set
   3. $\{\vec{x} \in \mathbb{R}^n : T(\vec{x}) = \vec{0}\}$
   4. $\{\vec{x} \in \mathbb{R}^n : A\vec{x} = \vec{0}\}$
   5. $\{A\vec{x} : \vec{x} \in \mathbb{R}^n\}$
   6. $\{T(\vec{x}) : \vec{x} \in \mathbb{R}^n\}$
   7. $\text{span}\{\vec{v}_1, \ldots, \vec{v}_n\}$ where $\vec{v}_i$ is the ith row of A
   8. $\det(A) \neq 0$
   9. the number of vectors in a basis for a subspace
   10. the number of vectors in a basis for the row space
   11. the number of vectors in a basis for the null space

2. Consider the following matrix:

$$A = \begin{pmatrix} 1 & 2 & -1 & -2 \\ 2 & 4 & -3 & -5 \\ -1 & -2 & 0 & 1 \end{pmatrix}$$

(a) (4 points) Give a basis for null(A).

Solution: The RREF of A is

$$\begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The general solution of the corresponding homogeneous system is then

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2s_1 + s_2 \\ s_1 \\ -s_2 \\ s_2 \end{pmatrix}$$

so that a basis for the null space is given by

$$\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}.$$

(b) (3 points) Give a basis for row(A).

Solution: The non-zero rows of the RREF of A are a basis for the row space, namely

$$\left\{ \begin{pmatrix} 1 & 2 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 1 \end{pmatrix} \right\}.$$

(c) (3 points) Give a basis for col(A).

Solution: The columns of A for which that column has a pivot in the RREF give a basis, namely

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} \right\}.$$

3. (a) (3 points) Compute

$$\det \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 \end{pmatrix}$$

Solution: 4.

(b) (2 points) Give an example of a non-invertible matrix of rank 2. You do not need to justify why your example works.

Solution: The matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

clearly has rank 2 but is not square so can't be invertible.

(c) (2 points) Suppose A and B are $n \times n$ matrices where $A = (I+B)(I-B)^{-1}$. Solve for B in terms of A; assume any inverses you encounter exist.

Solution: Multiplying on the right by $I - B$ gives $A - AB = I + B$, so that $A - I = AB + B = (A+I)B$, so that $B = (A+I)^{-1}(A-I)$.

(d) (3 points) Give an example of a linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ with one-dimensional range. You do not need to justify why your example works. What must the <u>dimension of the kernel</u> be?

Solution: Projecting onto the x-axis is such a linear transformation. Explicitly, this is

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ 0 \end{bmatrix}.$$

It has one-dimensional kernel, namely the y-axis. By the rank-nullity theorem, any such T must have one-dimensional kernel.